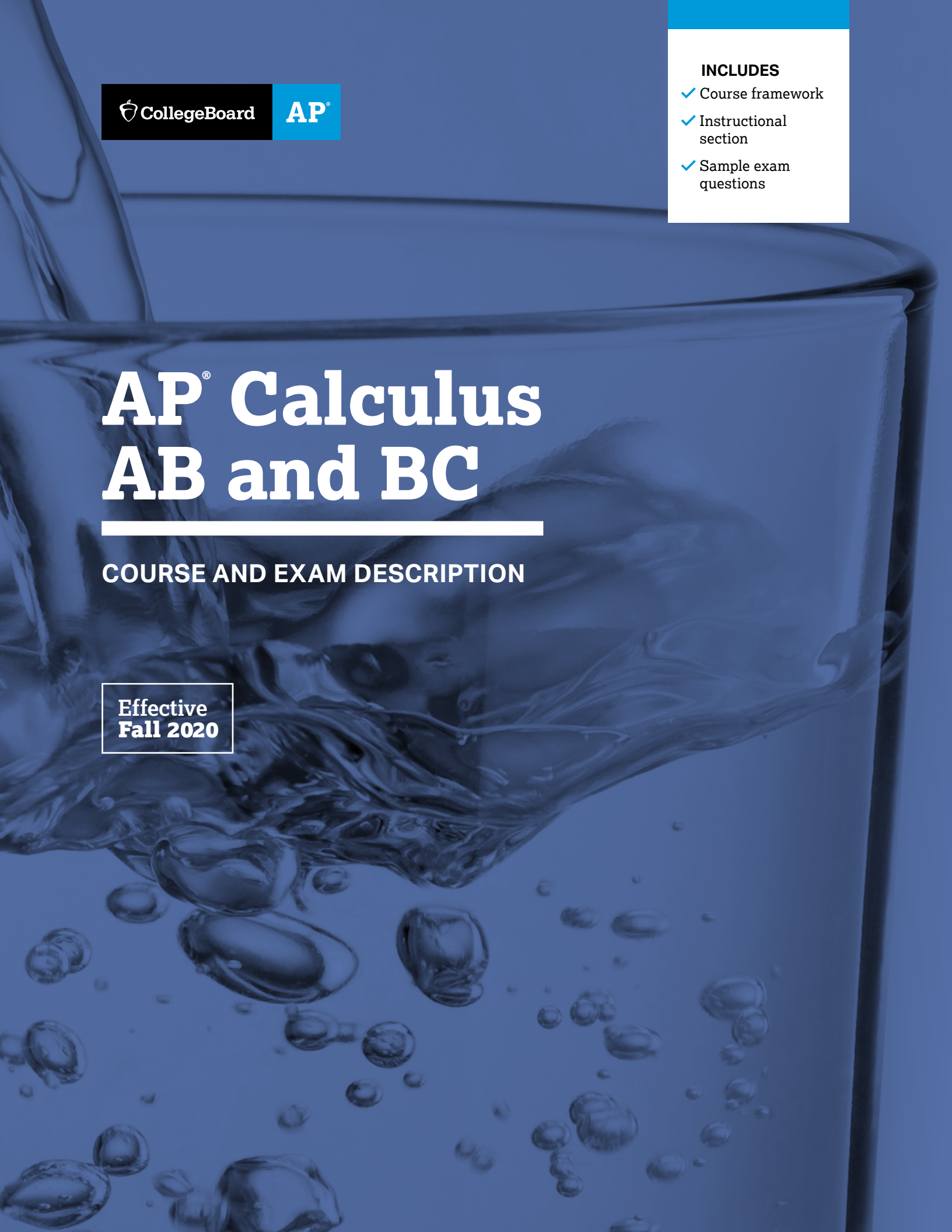

CollegeBoard AP

INCLUDES

- ✓ Course framework
- ✓ Instructional section
- ✓ Sample exam questions

AP® Calculus AB and BC

COURSE AND EXAM DESCRIPTION

Effective **Fall 2020**

AP® Calculus AB and BC

COURSE AND EXAM DESCRIPTION

Effective
Fall 2020

AP COURSE AND EXAM DESCRIPTIONS ARE UPDATED PERIODICALLY

Please visit AP Central (apcentral.collegeboard.org) to determine whether a more recent course and exam description is available.

Introduction

The course frameworks for AP Calculus AB and AP Calculus BC define content students must know and skills students must master in order to earn transferable, long-term understandings of calculus. The frameworks reflect a commitment to what college faculty value and mirror the curricula in corresponding college courses. Teachers may adjust the frameworks to meet state and local requirements.

The frameworks are organized into logical sequences, based on teacher input and commonly used textbooks. These sequences represent one reasonable learning pathway for each course, among many. Teachers may adjust the suggested sequencing of units or topics, although they will want to carefully consider how to account for such changes as they access course resources for planning, instruction, and assessment.

Balancing guidance and flexibility, this approach helps to prepare students for college credit and placement.

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Course Framework Components

Overview

This course framework provides a clear and detailed description of the course requirements necessary for student success. The framework specifies what students should know, be able to do, and understand to qualify for college credit or placement.

The course framework includes two essential components:

1 MATHEMATICAL PRACTICES

The mathematical practices are central to the study and practice of calculus. Students should develop and apply the described skills on a regular basis over the span of the course.

2 COURSE CONTENT

The course content is organized into commonly taught units of study that provide a suggested sequence for the course. These units comprise the content and conceptual understandings that colleges and universities typically expect students to master to qualify for college credit and/or placement. This content is grounded in big ideas, which are cross-cutting concepts that build conceptual understanding and spiral throughout the course.

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AP CALCULUS AB AND BC

Mathematical Practices

The AP Calculus AB and BC mathematical practices describe what a student should be able to do while exploring course concepts. The table that follows presents these practices, which students should develop during the AP Calculus AB and AP Calculus BC courses. These practices are categorized into skills, which form the basis of the tasks on the AP Exam.

The unit guides later in this publication embed and spiral these skills throughout the course, providing teachers with one way to integrate the skills in the course content with sufficient repetition to prepare students to transfer those skills when taking the AP Exam. Course content may be paired with a variety of skills on the AP Exam.

More detailed information about teaching the mathematical practices can be found in the Instructional Approaches section of this publication.

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AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

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AP CALCULUS AB AND BC

Course Content

Based on the Understanding by Design® (Wiggins and McTighe) model, this course framework provides a clear and detailed description of the course requirements necessary for student success. The framework specifies what students must know, be able to do, and understand, with a focus on big ideas that encompass core principles, theories, and processes of the discipline. The framework also encourages instruction that prepares students for advanced coursework in mathematics or other fields engaged in modeling change (e.g., pure sciences, engineering, or economics) and for creating useful, reasonable solutions to problems encountered in an ever-changing world.

Big Ideas

The big ideas serve as the foundation of the course and allow students to create meaningful connections among concepts. They are often abstract concepts or themes that become threads that run throughout the course. Revisiting the big ideas and applying them in a variety of contexts allows students to develop deeper conceptual understanding. Below are the big ideas of the course and a brief description of each.

BIG IDEA 1: CHANGE (CHA)

Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus—a central idea in AP Calculus.

BIG IDEA 2: LIMITS (LIM)

Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in calculus: for example, continuity, differentiation, integration, and series **BC ONLY**.

BIG IDEA 3: ANALYSIS OF FUNCTIONS (FUN)

Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others.

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UNITS

The course content is organized into commonly taught units. The units have been arranged in a logical sequence frequently found in many college courses and textbooks.

The eight units in AP Calculus AB and ten units in AP Calculus BC, and their weighting on the multiple-choice section of the AP Exam, are listed on the following page.

Pacing recommendations at the unit level and on the Course at a Glance provide suggestions for how teachers can teach the required course content and administer the Personal Progress Checks.

The suggested class periods are based on a schedule in which the class meets five days a week for 45 minutes each day. While these recommendations have been made to aid planning, teachers are of course free to adjust the pacing based on the needs of their students, alternate schedules (e.g., block scheduling), or their school's academic calendar.

TOPICS

Each unit is broken down into teachable segments called topics. The topic pages (starting on p. 35) contain the required content for each topic. Although most topics can be taught in one or two class periods, teachers should pace the course to suit the needs of their students and school.

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Exam Weighting for the Multiple-Choice Section of the AP Exam

Units	Exam Weighting (AB)	Exam Weighting (BC)
Unit 1: Limits and Continuity	**10–12%**	**4–7%**
Unit 2: Differentiation: Definition and Fundamental Properties	**10–12%**	**4–7%**
Unit 3: Differentiation: Composite, Implicit, and Inverse Functions	**9–13%**	**4–7%**
Unit 4: Contextual Applications of Differentiation	**10–15%**	**6–9%**
Unit 5: Analytical Applications of Differentiation	**15–18%**	**8–11%**
Unit 6: Integration and Accumulation of Change	**17–20%**	**17–20%**
Unit 7: Differential Equations	**6–12%**	**6–9%**
Unit 8: Applications of Integration	**10–15%**	**6–9%**
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions **BC ONLY**		**11–12%**
Unit 10: Infinite Sequences and Series **BC ONLY**		**17–18%**

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Spiraling the Big Ideas

The following table shows how the big ideas spiral across units.

Big Ideas	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
	Limits and Continuity	Differentiation: Definition and Fundamental Properties	Differentiation: Composite, Implicit, and Inverse Functions	Contextual Applications of Differentiation	Analytical Applications of Differentiation
Change CHA	✓	✓		✓	
Limits LIM	✓	✓		✓	
Analysis of Functions FUN	✓	✓	✓		✓

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Spiraling the Big Ideas *(cont'd)*

Big Ideas	Unit 6 Integration and Accumulation of Change	Unit 7 Differential Equations	Unit 8 Applications of Integration	Unit 9 Parametric Equations, Polar Coordinates, and Vector-Valued Functions BC ONLY	Unit 10 Infinite Sequences and Series BC ONLY
Change CHA	✓		✓	✓	
Limits LIM	✓				✓
Analysis of Functions FUN	✓	✓		✓	

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Course at a Glance

Plan

The Course at a Glance provides a useful visual organization of the AP Calculus AB and AP Calculus BC curricular components, including:

- Sequence of units, along with approximate weighting and suggested pacing. Please note, pacing is based on 45-minute class periods, meeting five days each week for a full academic year.
- Progression of topics within each unit.
- Spiraling of the big ideas and mathematical practices across units.

Teach

MATHEMATICAL PRACTICES

Mathematical practices spiral throughout the course.

- 1 Implementing Mathematical Processes
- 2 Connecting Representations
- 3 Justification
- 4 Communication and Notation

BIG IDEAS

Big ideas spiral across topics and units.

- CHA Change
- LIM Limits
- FUN Analysis of Functions

BC ONLY

The purple shading represents BC only content.

Assess

Assign the Personal Progress Checks—either as homework or in class—for each unit. Each Personal Progress Check contains formative multiple-choice and free-response questions. The feedback from the Personal Progress Checks shows students the areas where they need to focus.

UNIT 1 Limits and Continuity

AP EXAM WEIGHTING	10–12% AB 4–7% BC
CLASS PERIODS	~22–23 AB ~13–14 BC

CHA 2	1.1	Introducing Calculus: Can Change Occur at an Instant?
LIM 2	1.2	Defining Limits and Using Limit Notation
LIM 2	1.3	Estimating Limit Values from Graphs
LIM 2	1.4	Estimating Limit Values from Tables
LIM 1	1.5	Determining Limits Using Algebraic Properties of Limits
LIM 1	1.6	Determining Limits Using Algebraic Manipulation
LIM 1	1.7	Selecting Procedures for Determining Limits
LIM 3	1.8	Determining Limits Using the Squeeze Theorem
LIM 2	1.9	Connecting Multiple Representations of Limits
LIM 3	1.10	Exploring Types of Discontinuities
LIM 3	1.11	Defining Continuity at a Point
LIM 1	1.12	Confirming Continuity over an Interval
LIM 1	1.13	Removing Discontinuities
LIM 3	1.14	Connecting Infinite Limits and Vertical Asymptotes
LIM 2	1.15	Connecting Limits at Infinity and Horizontal Asymptotes
FUN 3	1.16	Working with the Intermediate Value Theorem (IVT)

Personal Progress Check 1

Multiple-choice: ~45 questions
Free-response: 3 questions (partial)

UNIT 2 Differentiation: Definition and Basic Derivative Rules

AP EXAM WEIGHTING	10–12% AB 4–7% BC
CLASS PERIODS	~13–14 AB ~9–10 BC

CHA 2	2.1	Defining Average and Instantaneous Rates of Change at a Point
CHA 1 4	2.2	Defining the Derivative of a Function and Using Derivative Notation
CHA 1	2.3	Estimating Derivatives of a Function at a Point
FUN 3	2.4	Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist
FUN 1	2.5	Applying the Power Rule
FUN 1	2.6	Derivative Rules: Constant, Sum, Difference, and Constant Multiple
FUN LIM 1	2.7	Derivatives of cos x, sin x, e^x, and ln x
FUN 1	2.8	The Product Rule
FUN 1	2.9	The Quotient Rule
FUN 1	2.10	Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions

Personal Progress Check 2

Multiple-choice: ~30 questions
Free-response: 3 questions (partial)

NOTE: Partial versions of the free-response questions are provided to prepare students for more complex, full questions that they will encounter on the AP Exam.

UNIT 3 Differentiation: Composite, Implicit, and Inverse Functions

AP EXAM WEIGHTING	9–13% AB 4–7% BC
CLASS PERIODS	~10–11 AB ~8–9 BC

- FUN 1 — 3.1 The Chain Rule
- FUN 1 — 3.2 Implicit Differentiation
- FUN 3 — 3.3 Differentiating Inverse Functions
- FUN 1 — 3.4 Differentiating Inverse Trigonometric Functions
- FUN 1 — 3.5 Selecting Procedures for Calculating Derivatives
- FUN 1 — 3.6 Calculating Higher-Order Derivatives

Personal Progress Check 3

Multiple-choice: ~15 questions
Free-response: 3 questions
(partial/full)

UNIT 4 Contextual Applications of Differentiation

AP EXAM WEIGHTING	10–15% AB 6–9% BC
CLASS PERIODS	~10–11 AB ~6–7 BC

- CHA 1 — 4.1 Interpreting the Meaning of the Derivative in Context
- CHA 1 — 4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration
- CHA 2 — 4.3 Rates of Change in Applied Contexts Other Than Motion
- CHA 1 — 4.4 Introduction to Related Rates
- CHA 3 — 4.5 Solving Related Rates Problems
- CHA 1 — 4.6 Approximating Values of a Function Using Local Linearity and Linearization
- LIM 3 — 4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms

Personal Progress Check 4

Multiple-choice: ~15 questions
Free-response: 3 questions

UNIT 5 Analytical Applications of Differentiation

AP EXAM WEIGHTING	15–18% AB 8–11% BC
CLASS PERIODS	~15–16 AB ~10–11 BC

- FUN 3 — 5.1 Using the Mean Value Theorem
- FUN 3 — 5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points
- FUN 2 — 5.3 Determining Intervals on Which a Function Is Increasing or Decreasing
- FUN 3 — 5.4 Using the First Derivative Test to Determine Relative (Local) Extrema
- FUN 1 — 5.5 Using the Candidates Test to Determine Absolute (Global) Extrema
- FUN 2 — 5.6 Determining Concavity of Functions over Their Domains
- FUN 3 — 5.7 Using the Second Derivative Test to Determine Extrema
- FUN 2 — 5.8 Sketching Graphs of Functions and Their Derivatives
- FUN 2 — 5.9 Connecting a Function, Its First Derivative, and Its Second Derivative
- FUN 2 — 5.10 Introduction to Optimization Problems
- FUN 3 — 5.11 Solving Optimization Problems
- FUN 1 3 — 5.12 Exploring Behaviors of Implicit Relations

Personal Progress Check 5

Multiple-choice: ~35 questions
Free-response: 3 questions

UNIT 6 Integration and Accumulation of Change

AP EXAM WEIGHTING: 17–20% AB 17–20% BC

CLASS PERIODS: ~18–20 AB ~15–16 BC

CHA 4	6.1	Exploring Accumulations of Change
LIM 1	6.2	Approximating Areas with Riemann Sums
LIM 2	6.3	Riemann Sums, Summation Notation, and Definite Integral Notation
FUN 1	6.4	The Fundamental Theorem of Calculus and Accumulation Functions
FUN 2	6.5	Interpreting the Behavior of Accumulation Functions Involving Area
FUN 3	6.6	Applying Properties of Definite Integrals
FUN 3	6.7	The Fundamental Theorem of Calculus and Definite Integrals
FUN 4	6.8	Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation
FUN 1	6.9	Integrating Using Substitution
FUN 1	6.10	Integrating Functions Using Long Division and Completing the Square
FUN 1	6.11	Integrating Using Integration by Parts BC ONLY
FUN 1	6.12	Using Linear Partial Fractions BC ONLY
LIM 1	6.13	Evaluating Improper Integrals BC ONLY
FUN 1	6.14	Selecting Techniques for Antidifferentiation

Personal Progress Check 6

Multiple-choice:
- ~25 questions (AB)
- ~35 questions (BC)

Free-response: 3 questions

UNIT 7 Differential Equations

AP EXAM WEIGHTING: 6–12% AB 6–9% BC

CLASS PERIODS: ~8–9 AB ~9–10 BC

FUN 2	7.1	Modeling Situations with Differential Equations
FUN 3	7.2	Verifying Solutions for Differential Equations
FUN 2	7.3	Sketching Slope Fields
FUN 4	7.4	Reasoning Using Slope Fields
FUN 1	7.5	Approximating Solutions Using Euler's Method BC ONLY
FUN 1	7.6	Finding General Solutions Using Separation of Variables
FUN 1	7.7	Finding Particular Solutions Using Initial Conditions and Separation of Variables
FUN 3	7.8	Exponential Models with Differential Equations
FUN 3	7.9	Logistic Models with Differential Equations BC ONLY

Personal Progress Check 7

Multiple-choice:
- ~15 questions (AB)
- ~20 questions (BC)

Free-response: 3 questions

UNIT 8 Applications of Integration

AP EXAM WEIGHTING: 10–15% AB 6–9% BC

CLASS PERIODS: ~19–20 AB ~13–14 BC

CHA 1	8.1	Finding the Average Value of a Function on an Interval
CHA 1	8.2	Connecting Position, Velocity, and Acceleration of Functions Using Integrals
CHA 3	8.3	Using Accumulation Functions and Definite Integrals in Applied Contexts
CHA 4	8.4	Finding the Area Between Curves Expressed as Functions of x
CHA 1	8.5	Finding the Area Between Curves Expressed as Functions of y
CHA 2	8.6	Finding the Area Between Curves That Intersect at More Than Two Points
CHA 3	8.7	Volumes with Cross Sections: Squares and Rectangles
CHA 3	8.8	Volumes with Cross Sections: Triangles and Semicircles
CHA 3	8.9	Volume with Disc Method: Revolving Around the x- or y-Axis
CHA 2	8.10	Volume with Disc Method: Revolving Around Other Axes
CHA 4	8.11	Volume with Washer Method: Revolving Around the x- or y-Axis
CHA 2	8.12	Volume with Washer Method: Revolving Around Other Axes
CHA 3	8.13	The Arc Length of a Smooth, Planar Curve and Distance Traveled BC ONLY

Personal Progress Check 8

Multiple-choice: ~30 questions
Free-response: 3 questions

UNIT 9 Parametric Equations, Polar Coordinates, and Vector-Valued Functions BC ONLY

AP EXAM WEIGHTING	N/A AB	11–12% BC
CLASS PERIODS	N/A AB	~10–11 BC

CHA 2	9.1	Defining and Differentiating Parametric Equations
CHA 1	9.2	Second Derivatives of Parametric Equations
CHA 1	9.3	Finding Arc Lengths of Curves Given by Parametric Equations
CHA 1	9.4	Defining and Differentiating Vector-Valued Functions
FUN 1	9.5	Integrating Vector-Valued Functions
FUN 1	9.6	Solving Motion Problems Using Parametric and Vector-Valued Functions
FUN 2	9.7	Defining Polar Coordinates and Differentiating in Polar Form
CHA 3	9.8	Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve
CHA 3	9.9	Finding the Area of the Region Bounded by Two Polar Curves

Personal Progress Check 9

Multiple-choice: ~25 questions
Free-response: 3 questions

UNIT 10 Infinite Sequences and Series BC ONLY

AP EXAM WEIGHTING	N/A AB	17–18% BC
CLASS PERIODS	N/A AB	~17–18 BC

LIM 3	10.1	Defining Convergent and Divergent Infinite Series
LIM 3	10.2	Working with Geometric Series
LIM 3	10.3	The nth Term Test for Divergence
LIM 3	10.4	Integral Test for Convergence
LIM 3	10.5	Harmonic Series and p-Series
LIM 3	10.6	Comparison Tests for Convergence
LIM 3	10.7	Alternating Series Test for Convergence
LIM 3	10.8	Ratio Test for Convergence
LIM 3	10.9	Determining Absolute or Conditional Convergence
LIM 1	10.10	Alternating Series Error Bound
LIM 3 2	10.11	Finding Taylor Polynomial Approximations of Functions
LIM 1	10.12	Lagrange Error Bound
LIM 2	10.13	Radius and Interval of Convergence of Power Series
LIM 2	10.14	Finding Taylor or Maclaurin Series for a Function
LIM 3	10.15	Representing Functions as Power Series

Personal Progress Check 10

Multiple-choice: ~45 questions
Free-response: 3 questions